When sparsity and low-rank meet: ALOHA!

INTERVIEW BY **XIN MIAO**

EDITOR'S PICK FOR DECEMBER

Dongwook Lee is currently a Ph.D. student at the Korea Advanced Institute of Science and Technology (KAIST). He works on advanced image reconstruction techniques for dynamic MRI. His paper, selected as the Editor's pick for December, is entitled "Acceleration of MR Parameter Mapping Using Annihilating Filter-based Low Rank Hankel Matrix (ALOHA)." ALOHA is a novel image reconstruction algorithm with the goal of clear, artifact free images acquired from very fast imaging schemes. For this paper, ALOHA was applied to accelerated MR parameter mapping, but could also be used for dynamic and parallel MRI, and even non-MR applications. We recently invited Dongwook and his supervisor, Dr. Jong Chul Ye, to talk about this paper.

> *We realized that the killer application of ALOHA should be accelerated MRI.*
>
> –Jong Chul Ye

MRMH: Can you tell us a little about your academic background?

Dongwook: I'm a third-year Ph.D. student at KAIST. I also did my bachelor's and master's degrees here.

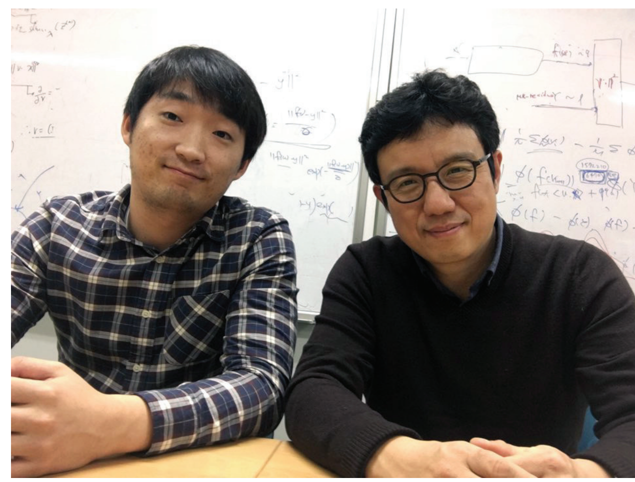
Jong: I am currently KAIST endowed chair and a professor at the Department of Bio and Brain Engineering. For the past 11 years at KAIST, my research has been focused on compressed sensing (CS) image reconstruction, signal processing, and machine learning for medical imaging applications such as MRI and CT.

MRMH: ALOHA is an interesting name. How did you come up with it?

Jong: Regarding the name "ALOHA", it stands for Annihilating filter-based LOw-rank Hankel matrix Approach. Looking back to when we discovered the algorithm, I asked my student to invent a nice name, so that people could easily remember it. The student brought me several candidates, but as soon as I saw this name, I immediately thought "ALOHA" is perfect because it has all the important terms, and moreover it gives very positive feelings. What a coincidence that the coming ISMRM will be held on ALOHA island!

MRMH: Can you explain the main idea and inspiration behind this paper?

Dongwook: Compressed sensing (CS) tries to reconstruct an image from sub-Nyquist sampling by exploiting the sparsity of the image in certain transform domains. The main idea of ALOHA is converting the CS problem to a weighted k-space interpolation problem. In ALOHA, a structured matrix, called Hankel matrix, is constructed from weighted k-space data. If the MRI images are sparse in a certain transform domain, such as total variation or wavelet, we expect the corresponding Hankel matrix to be rank-deficient. Based on this theory, image reconstruction is formulated as a low-rank matrix completion problem.



Dongwook Lee and Jong Chul Ye.

Jong: The original form of ALOHA occurred through serendipity. My former student, Kyong Hwan Jin, was testing various interpolation approaches for optical microscopy (not MRI). He showed me excellent interpolation results using structured matrix completion. We spent several months to establish the mathematical framework, which gave birth to the theory behind ALOHA. Then we realized that the killer application of ALOHA should be accelerated MRI.

MRMH: Can you explain the dual relationship between sparsity and low-rankness?

Dongwook: This duality comes from basic signal processing: the multiplication in spatial domain corresponds to convolution in Fourier domain. If an image A is sparse in the spatial domain, then we can find an

Lee D, Jin KH, Kim EY, Park SH, Ye JC. Acceleration of MR parameter mapping using annihilating filter-based low rank hankel matrix (ALOHA). *Magn Reson Med.* 2016;76:1848–1864. doi: 10.1002/mrm.26081
http://onlinelibrary.wiley.com/doi/10.1002/mrm.26081/full

annihilating function B to make the product of A and B equal zero. Equivalently, in Fourier domain, we can build a discrete convolution matrix from the Fourier transform of image A. This convolution matrix should have low rank because the multiplication of it with an annihilating filter is zero.

Jong: The images don't have to be sparse in the spatial domain. If they can be sparsified using some transformation, then we can apply an appropriate k-space weighting based upon this transformation. After the weighting, we are able to interpret sparsity in the image domain as low-rankness in the Fourier domain.

MRMH: What's the biggest difference between ALOHA and other low-rank based methods, like kt-SLR and LORAKS?

Jong: SAKE, LORAKS, and ALOHA arrange k-space data into a structured Hankel matrix. These methods all rely on the low-rank property of the Hankel matrix to reconstruct images. SAKE exploits multi-coil correlation, LORAKS exploits finite spatial-support or smooth phase condition, and ALOHA reformulates sparsity in the transform domain as low-rankness in Fourier domain. Before forming the Hankel matrix, ALOHA applies a weighting to k-space data, based on the chosen sparsifying transform. In this way, ALOHA can be applied in more general situations where a suitable sparsifying transform is found. In addition, ALOHA also allows data redundancy in any dimension to be exploited in the same framework. For example, for dynamic imaging applications, ALOHA constructs the Hankel matrix from k-t space data. To integrate parallel imaging, ALOHA stacks multi-channel data side by side.

Dongwook: While kt-SLR also exploits low rank properties, it is different from ALOHA-like methods because its matrices do not have Hankel-like structure. Kt-SLR stacks vectorized dynamic images from all time frames with high temporal correlation to create a low-rank matrix. Note that kt-SLR keeps the original structure of the matrix without lifting to a Hankel-like structure.

MRMH: Can you talk about the advantages of ALOHA compared to existing CS methods?

Dongwook: The most important advantage of ALOHA is accuracy. ALOHA's annihilating filter exploits the edge information, so the edges are reconstructed reliably. Reconstruction error appears as random noise rather than structured noise along the edges in conventional CS. This could be an important advantage in clinical applications.

MRMH: Why is parameter mapping your application of choice for ALOHA?

Dongwook: If you look at any T_1 and T_2 dataset, contrast changes over time but structure is maintained, indicating sparsity in the x-f space. Because of this sparsity, we can construct a rank-deficient Hankel matrix from weighted k-t space measurements.

Jong Chul Ye, Juyoung Lee (Ph.D. student), and Dongwook Lee at Korea Advanced Institute of Science & Technology.

Jong: Parameter mapping demonstrates advantages of ALOHA over existing methods. We have previously tried global and locally low-rank approaches on parameter mapping. However, above a certain acceleration factor, global and locally low-rank methods leave unresolved aliasing artifacts, which can indicate a need for constraints beyond the low-rankness due to temporal correlation. Therefore, we wanted to further explore the low-rankness resulting from x-f sparsity using ALOHA.

MRMH: I'm sure ALOHA is not limited to parameter mapping. Is there any other application where ALOHA might be helpful?

Dongwook: Yes. Another interesting application of ALOHA is Nyquist ghost correction in EPI, which is the research work of my colleagues, Juyoung Lee and Kyong Hwan Jin. The mismatch between odd and even lines causes Nyquist ghosting artifact in EPI. We have proposed that such artifacts can be corrected by solving a k-space interpolation problem using ALOHA. Specifically, the odd and even lines were separated from the EPI dataset and stacked side by side in the form of a Hankel matrix. Due to the high correlation between the odd and even samples, this Hankel matrix should be rank-deficient. This work was also published in the current issue of MRM.

Jong: We have applied ALOHA for dynamic MRI, parallel MRI as well as MR artifact correction. ALOHA can even be used for non-MR image processing applications such as image inpainting and super resolution microscopy. ■

ALOHA's annihilating filter exploits the edge information, so the edges are reconstructed reliably.

–Dongwook Lee